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The theory of heating and evaporation of a spherical particle exposed to optical radiation

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Abstract—A theoretical investigation into the processes of heating and evaporation of a spherical particle under strong optical radiation is carried out. A system of equations is formulated which describe the process of particle evaporation in diffusional regime. Based on numerical solution of this system of equations, the evaporation of air-suspended water droplets under the i.r. optical radiation is investigated in detail. The time-dependent and stationary values of the evaporation process parameters have been obtained for a wide range of the initial problem parameters. The calculated results are compared with the experimental data. A simple model of particle evaporation is considered, analytical and numerical solutions have been obtained.

1. INTRODUCTION

THE HEATING and evaporation of spherical solid and liquid particles under the influence of optical radiation have been investigated theoretically from 1965 in a number of studies [1–14], where a wide range of problems were considered with the use of different approximations. Different regimes of regular evaporation of particles under the effect of optical radiation, i.e. diffusional, convective, gas dynamic, were considered [1–7, 10, 12] and the regions of their development were established. The time dependences of the radius r_0 and temperature T_0 of evaporating particles were calculated numerically [4–8, 10, 13]. Approximate analytical solutions were obtained for the dependence of r_0, T_0 on time t in the case of quasistationary evaporation [2, 5]. The effect of the condensation coefficient α and optical constants of the particle substance on the evaporation process parameters was studied in works [10, 11]. Common to all of these works [1–14] are:

(1) No allowance for the dependence of the problem thermophysical parameters (diffusion coefficient D, thermal conductivities κ_1, κ_2, heat of evaporation L) on temperature, except for works [9, 10, 12, 13]. However, the experimental dependences of D, κ_1, κ_2, L on temperature in these studies differ substantially from those reported in reference [15].

(2) No allowance for the vapour density and temperature jumps near the evaporating particle surface, except for works [10, 12]. In reference [12] this has been done with the aid of approximate phenomenological coefficients.

(3) Except for works [10, 11], numerical and analytical calculations of the evaporation process are based on the approximate factor of effectiveness of optical radiation absorption by a particle $K_{ab}(r_0)$ [2] which leads to an error of about 30%.

(4) No coverage of the energy response characteristics of the evaporation process for particles with different initial radii r_∞.

(5) The lack of comparison between theoretical and experimental results for particles having characteristic dimensions of about 10 μm.

The analysis of these problems is of great interest and is conducted in this paper. Accounting for the actual dependences of the problem thermophysical parameters on temperature and on density and temperature jumps near the surface of a particle, a system of equations is formulated which correctly describes the process of particle evaporation in diffusional regime on exposure to optical radiation. The effectiveness factor $K_{ab}(r_0)$ [10, 11] is used which incurs an error of no more than 1%. Based on numerical computer solution of this system of equations, the dependence of the evaporation process parameters on the optical radiation energy flux density I_0, initial particle radius r_∞ and temperature T_∞ evaporation coefficient α and optical constants n_λ, κ_λ of the particle substance is investigated. Some of the results obtained are compared with the experimental data; the non-stationary stage of evaporation is considered. Approximate analytical solutions are obtained for $r_0(t)$ and $T_0(t)$ in non-stationary evaporation, and the estimates of the accuracy and regions of applicability of widely used approximations are made. The results of the present work are of practical interest for the investigation of heat and mass transfer in dispersed media, optical radiation propagation in a medium with particles, radiation-produced bleaching of polydispersed water aerosol.

NOMENCLATURE

c_1	heat capacity of gas medium $[\mathrm{JK^{-1}\,kg^{-1}}]$
c_2	heat capacity of particle substance per molecule $[\mathrm{JK^{-1}}]$
D	diffusion coefficient of particle substance vapour in surrounding gas $[\mathrm{m^2\,s^{-1}}]$
E_T	thermal energy of particle
I_0	radiation energy flux density $[\mathrm{Wm^{-2}\,s^{-1}}]$
j_m	mass flux density
j_ε	energy flux density
j_T	density of energy flux dissipated by conduction
K_ab	effectiveness factor of radiation absorption by particle
K_sc	effectiveness factor of radiation scattering by particle
k	Boltzmann constant, $1.38054 \times 10^{-23}\,\mathrm{JK^{-1}}$
L	evaporation heat of one molecule of particle substance [J]
l	free path length of vapour molecule in surrounding gas
m	mass of vapour molecule [kg]
n_λ	refractive index of particle substance at wave length λ
n	number density of molecules of particle substance vapour $[\mathrm{m^{-3}}]$
$\bar{n}_0$	number density of molecules (atoms) of particle substance $[\mathrm{m^{-3}}]$
q	power density of heat sources in particle
r_0	instantaneous particle radius $[\mu\mathrm{m}]$
r	radial coordinate [m]
S_0	particle surface area $[\mathrm{m^2}]$
T	temperature [K]
T_0	particle temperature [K]
t	time [s]
V_0	particle volume $[\mathrm{m^3}]$
x	dimensionless particle radius
y	dimensionless particle temperature.

Greek symbols

α	condensation (evaporation) coefficient
κ_λ	absorption coefficient of particle substance at wavelength λ
κ_1	thermal conductivity of gas medium $[\mathrm{Wm^{-1}\,K^{-1}}]$
κ_2	thermal conductivity of particle substance $[\mathrm{Wm^{-1}\,K^{-1}}]$
λ	optical radiation wavelength
ρ_1	gas medium density
τ	dimensionless time
χ_1	thermal diffusivity of gas medium $[\mathrm{m^2\,s^{-1}}]$
χ_2	thermal diffusivity of particle substance $[\mathrm{m^2\,s^{-1}}]$.

Subscripts

∞	initial value
m	maximum.

2. PROBLEM FORMULATION AND SYSTEMS OF EQUATIONS

Let, starting from time $t = 0$, a spherical particle of initial radius r_∞ and temperature T_∞ (equal to the surrounding medium temperature) be exposed to optical radiation with the wavelength λ and energy flux density $I_0(t)$. The particle absorbs the energy of the incident optical radiation, becomes heated and starts to exchange heat with the surrounding gas medium. First, the energy is given off to the surrounding medium by heat conduction and then, with an increase of the surface temperature T_0, by evaporation. The processes of heating and heat and mass transfer of the spherical particle are described by equation (1), subject to initial and boundary conditions (2) in the spherical coordinate system with the origin at the particle centre, as

$$\bar{n}_0 c_2 \frac{\partial T}{\partial t} = \mathrm{div}\,(\kappa_2\,\mathrm{grad}\,T) + q \qquad (1)$$

$$-\kappa_2\,\mathrm{grad}\,T|_{r=r_0} = j_\varepsilon \qquad (2)$$

$$r_0(t=0) = r_\infty, \quad T(r, t=0) = T_\infty, \quad r \leqslant r_0,$$

where j_ε is the density of the energy flux transferred by heat conduction and evaporation, q is the density of the energy release power in the particle due to radiation energy absorption. Generally, the energy release q can be a complicated function of coordinates and be substantially inhomogeneous [16]. The inhomogeneity degree depends on λ, r_0 and the optical constants of the particle substance; for particles with $r_0 \ll \lambda$, the q is practically homogeneous. After the start of energy release, the particle is heated non-uniformly and this leads to heat transfer by conduction and to equalization of temperatures inside the particle. In a liquid droplet, in addition to temperature field equalization due to heat conduction, non-uniform heating causes a thermocapillary circulation (mixing) of liquid and this also leads to equalization of the temperature field [17]. If the characteristic time of heat conduction-induced temperature field equilibration inside the particle, $t_\mathrm{T} \sim r_0^2/\chi_2$ (for particle and droplet), and that induced by thermocapillary liquid circulation, $t_\mathrm{c} \sim r_0^2/v_2$, where v_2 is the kinematic liquid viscosity (for a droplet), are much smaller than the characteristic times of particle heating and evaporation, then one may consider a uniformly heated particle (droplet) with a certain volume-averaged temperature T_0. Corresponding numerical estimates will be given below.

The system of equations, which describes the heating and evaporation of the particle and which results from equation (1) by integration over the volume for a spherically symmetric case, by passage to the averaged temperature T_0 and introduction of the evaporation kinetics equations, has the form

$$\bar{n}_0 c_2 V_0 \frac{\mathrm{d}T_0}{\mathrm{d}t} = \frac{1}{4} I_0(t) K_{\mathrm{ab}} S_0 - j_{\varepsilon} S_0 \tag{3}$$

$$\bar{n}_0 \frac{\mathrm{d}V_0}{\mathrm{d}t} = -j_{\mathrm{m}} S_0$$

where

$$\int_0^{r_0} q(t) 4\pi r^2 \, \mathrm{d}r = \tfrac{1}{4} I_0(t) K_{\mathrm{ab}} S_0,$$

j_{m} is the density of a flux of molecules (atoms) evaporating from the particle surface per unit time. For $K_{\mathrm{ab}}(r_0)$, the following approximation [10] will be used:

$$K_{\mathrm{ab}} = \bar{k}_{\mathrm{ab}} r_0 [1 - \mathrm{e}^{-a/r_0}] \tag{4}$$

where $\bar{k}_{\mathrm{ab}}$ and a are certain constants. Equation (4) approximates the tabulated data on $K_{\mathrm{ab}}(r_0)$ [18] for different radiation wavelengths within the range $0 < r_0 \leqslant 50$ μm with an error of about 1–5%.

2.1. *Diffusional regime of evaporation*

In the diffusional regime of particle evaporation, the heat and mass are transferred by heat conduction and diffusion. A corresponding system of equations for spherical symmetry is

$$\rho_1 c_1 \frac{\partial T}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r}\left(\kappa_1 r^2 \frac{\partial T}{\partial r}\right) \tag{5}$$

$$\frac{\partial n}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r}\left(D r^2 \frac{\partial n}{\partial r}\right)$$

with the temperature drops in the surrounding gas being assumed small, $(T_0 - T_\infty)/T_\infty < 1$, and the pressure constant. The external heat and mass transfer problem, equation (5), is characterized by the times for establishing the quasistationary profiles of temperature, $t_{\mathrm{T}}^1 \sim r_0^2/\chi_1$, and of vapour density, $t_{\mathrm{d}} \sim r_0^2/D$. Since the characteristic times of particle heating and evaporation are generally much greater than the characteristic times t_{T}^1 and t_{d}, then, instead of solving non-stationary equations (5), it is possible to pass over to the solution of the quasistationary equations

$$\frac{1}{r^2} \frac{\mathrm{d}}{\mathrm{d}r}\left(r^2 \kappa_1 \frac{\mathrm{d}T}{\mathrm{d}r}\right) = 0,$$

$$\frac{1}{r^2} \frac{\mathrm{d}}{\mathrm{d}r}\left(r^2 D \frac{\mathrm{d}n}{\mathrm{d}r}\right) = 0 \tag{6}$$

with boundary conditions

$$T(r_0) = \bar{T}, \quad T(r \to \infty) = T_\infty,$$

$$n(r_0) = \bar{n}, \quad n(r \to \infty) = n_\infty \tag{7}$$

where $\bar{T}$, $\bar{n}$, and T_∞, n_∞ are the values of T and n immediately at the particle surface and at infinity, respectively. In a wide temperature range it is possible, with fair accuracy, to represent κ_1 and D as power functions of temperature [15]:

$$\kappa_1 = \kappa_{1\infty}\left(\frac{T}{T_\infty}\right)^{p_1}, \quad D = D_\infty\left(\frac{T}{T_\infty}\right)^{b} \tag{8}$$

where $\kappa_{1\infty} = \kappa_1(T_\infty)$, $D_\infty = D(T_\infty)$, p_1 and b are certain constants. The inclusion of the temperature dependence of κ_1 and D is important, since the heat and mass transfer processes in the surrounding gas control the particle evaporation dynamics. The solution of the heat conduction equation in system (6), with equations (7) and (8) for temperature distribution in the space surrounding an evaporating particle taken into account, has the form

$$T = T_\infty \left[1 + \frac{r_0}{r}(\bar{T}_{\mathrm{N}}^{p_1+1} - 1)\right]^{1/(p_1+1)} \tag{9}$$

where $\bar{T}_{\mathrm{N}} = \bar{T}/T_\infty$. The density of the heat flux transferred from the particle surface by heat conduction is determined as

$$j_{\mathrm{T}} = -\kappa_1 \frac{\mathrm{d}T}{\mathrm{d}r}\bigg|_{r=r_0} = \frac{\kappa_{1\infty} T_\infty}{(p_1+1) r_0}(\bar{T}_{\mathrm{N}}^{p_1+1} - 1). \tag{10}$$

In the case of $p_1 - b = -1$, the vapour density distribution in the space around the evaporating particle is given by equation (6), with equations (8) and (9) taken into account, as [19]

$$n = n_\infty + \frac{(\bar{n} - n_\infty)}{(p_1+1)\ln \bar{T}_{\mathrm{N}}} \ln\left[1 + \frac{r_0}{r}(\bar{T}_{\mathrm{N}}^{p_1+1} - 1)\right] \tag{11}$$

while the density of the mass flux transferred from the droplet surface by diffusion is determined as

$$j_{\mathrm{m}} = -D \frac{\mathrm{d}n}{\mathrm{d}r}\bigg|_{r=r_0} = \frac{D_\infty}{(p_1+1) r_0}(\bar{T}_{\mathrm{N}}^{p_1+1} - 1)\frac{\bar{n} - n_\infty}{\ln \bar{T}_{\mathrm{N}}}. \tag{12}$$

In the case of $p_1 - b \neq -1$, the distribution n from equations (6), (8) and (9) is defined as

$$n = n_\infty + (\bar{n} - n_\infty) \times \frac{[\{1 + (r_0/r)(\bar{T}_{\mathrm{N}}^{p_1+1} - 1)\}^{\frac{p_1-b+1}{p_1+1}} - 1]}{\bar{T}_{\mathrm{N}}^{p_1-b+1} - 1} \tag{13}$$

while the density of the mass flux j_{m} is

$$j_{\mathrm{m}} = -D \frac{\mathrm{d}n}{\mathrm{d}r}\bigg|_{r=r_0} = \frac{D_\infty (p_1 - b + 1)}{r_0 (p_1+1)} \cdot \frac{(\bar{T}_{\mathrm{N}}^{p_1+1} - 1)}{(\bar{T}_{\mathrm{N}}^{p_1-b+1} - 1)}(\bar{n} - n_\infty). \tag{14}$$

The expressions obtained for the distribution of temperature (9), density (11) and (13), heat fluxes (10) and mass fluxes (12) and (14) seem to be the most general of those available in the literature. Specific cases of the temperature dependence of κ_1 and D, equation (8), have been considered in the works [9, 10, 12, 13]. For example, it was assumed that $p_1 = 0$, $b = 2$ [10]; $p_1 = 1/2$, $b = 3/2$ [9, 13]; $p_1 = 1/2$, $b = 2$ [12]. At the same time relations (9), (10) and (11)–(14) are valid for any $p_1 > 0$ and $b > 0$ and not only for integer or half-integer ones.

When κ_1 and D are independent of temperature, $\kappa_1 = \kappa_{1\infty} = \text{const.}$, $D = D_\infty = \text{const.}$, and $p_1 = 0$, $b = 0$, equations (9) and (13) yield the following linear quasistationary distributions for T and n:

$$T = T_\infty + \frac{r_0}{r}(\bar{T} - T_\infty), \quad n = n_\infty + \frac{r_0}{r}(\bar{n} - n_\infty) \tag{15}$$

and equations (10) and (16) give, for j_T and j_m,

$$j_T = \frac{\kappa_{1\infty}}{r_0}(\bar{T} - T_\infty) \tag{16}$$

$$j_m = \frac{D_\infty}{r_0}(\bar{n} - n_\infty)$$

which, when the density and temperature jumps at the particle surface are ignored, i.e. at $\bar{T} = T_0$, $\bar{n} = n_0$, coincide with the well-known expressions from reference [20].

The gas temperature $\bar{T}$ and vapour density $\bar{n}$ immediately at the particle surface are determined on the basis of the kinetic theory. The quantity $\bar{T}$ is given by the relation [21]

$$\bar{T} = T_0 - vl\left|\frac{dT}{dr}\right|_{r=r_0} \tag{17}$$

where

$$v = \frac{75\pi}{128}\,\frac{2 - 0.83\,a_1}{a_1};$$

a_1 is the accommodation coefficient assumed to be equal to unity, l the free path length of a molecule of the surrounding gas. With equation (9) taken into account, the expression for $\bar{T}$ has the form

$$\bar{T} \approx \left[T_\infty^{p_1+1} + \frac{T_0^{p_1+1} - T_\infty^{p_1+1}}{1 + [vl/(p_1+1)r_0]}\right]^{\frac{1}{p_1+1}}. \tag{18}$$

In order to determine $\bar{n}$, the expression for j_m is used which is obtained from the kinetic consideration of evaporation and condensation processes (the Knudsen equation) [22]

$$j_m = \alpha\left[n_0\left(\frac{kT_0}{2\pi m}\right)^{1/2} - \bar{n}\left(\frac{k\bar{T}}{2\pi m}\right)^{1/2}\right] \tag{19}$$

where α is the coefficient of evaporation (assumed to be equal to that of condensation) of the particle substance, n_0 is the saturated vapour density at the particle surface temperature T_0[15] and m the mass of a vapour molecule. Equations (13), (14), (21) and (15), (16), (19) finally yield:

at $p_1 = b - 1$

$$\bar{n} = \frac{n_0\alpha(kT_0/2\pi m)^{1/2} + (D_\infty n_\infty/r_0 b\ln\bar{T}_N)(\bar{T}_N^{p_1+1} - 1)}{\alpha(k\bar{T}/2\pi m)^{1/2} + (D_\infty/r_0 b\ln\bar{T}_N)(\bar{T}_N^{p_1+1} - 1)}$$

$$j_m = \frac{n_0 T_0^{1/2} - n_\infty\bar{T}^{1/2}}{(2\pi m/k\alpha^2)^{1/2} + (r_0\bar{T}^{1/2} b\ln\bar{T}_N)/D_\infty(\bar{T}_N^{p_1+1} - 1)} \tag{20}$$

at $p_1 \neq b - 1$

$$\bar{n} = \frac{n_0\alpha(kT_0/2\pi m)^{1/2} + [D_\infty n_\infty(p_1 - b + 1)/r_0(p_1+1)]\,[(\bar{T}_N^{p_1+1} - 1)/(\bar{T}_N^{p_1-b+1} - 1)]}{\alpha(k\bar{T}/2\pi m)^{1/2} + [D_\infty(p_1 - b + 1)/r_0(p_1+1)]\,[(\bar{T}_N^{p_1+1} - 1)/(\bar{T}_N^{p_1-b+1} - 1)]} \tag{21}$$

$$j_m = \frac{n_0 T_0^{1/2} - n_\infty\bar{T}^{1/2}}{(2\pi m/k\alpha^2)^{1/2} + \dfrac{r_0\bar{T}^{1/2}(p_1+1)(\bar{T}_N^{p_1-b+1} - 1)}{D_\infty(p_1 - b + 1)(\bar{T}_N^{p_1+1} - 1)}}.$$

The density of the energy flux j_ε from a particle evaporating in diffusional regime is determined by the relation

$$j_\varepsilon = j_T + j_m L(T_0), \tag{22}$$

where $L(T_0)$ is the heat of evaporation of one molecule of particle substance [15].

In reference [3, 4], the diffusional–convective regime of particle evaporation due to radiation is considered in the case when, along with the diffusional flux, the convective motion of the medium is allowed for in the constant pressure approximation. In work [5] the gas dynamic regime of particle evaporation with shock wave formation [23] was considered for which $j_\varepsilon = j_m L_\infty$ can be determined from

$$j_\varepsilon \approx \left(\frac{8}{\pi}\right)^{1/2} n_0\left(\frac{kT_0}{m}\right)^{1/2} \times \frac{(p_0/p_\infty) - 1}{(8m_g jT_0/\pi m T_\infty)^{1/2} + (4p_0/p_\infty) - 1}\cdot L_\infty \tag{23}$$

where p_0 is the pressure of saturated vapours at T_0, p_∞ the surrounding gas pressure, $j = c_p/c_v$ the specific heat ratio for a gas (air), c_p and c_v the heat capacities at constant pressure and constant volume per molecule;

the water vapour heat capacity c_p is taken to be $c_p = 4\,k$; m_g the mass of a gas molecule.

The solution of the system of equations (3) together with the expressions for j_m and j_ε and boundary conditions

$$r_0(t = 0) = r_\infty, \quad T_0(t = 0) = T_\infty \tag{24}$$

allows one to obtain the time dependences of the temperature and radius of an evaporating droplet.

2.2. *Parameters of the particle evaporation process*

Considerable interest attaches to the differential parameter

$$D_1(r_0) = \frac{I_0 K_{ab}(r_0)}{4 j_\varepsilon} \tag{25}$$

which can be termed the degree of the quasistationarity of the particle evaporation process [see equation (3)]. It represents the ratio of energy release in a particle to energy losses by evaporation and heat conduction and characterizes the imbalance between these quantities and the rate of change of the quantity T_0. Also the integral energy response characteristics will be considered which represent the particle evaporation process from the instant of exposure to radiation, $t = 0$, up to the time t considered, i.e. the radiation energies $Q_{ab}(r_0, t)$ and $Q_{sc}(r_0, t)$ respectively absorbed and scattered by the particle, the energies $Q_{ev}(r_0, t)$, $Q_T(r_0, t)$ expended for particle evaporation and removed from the particle by heat conduction

$$Q_{ab} = \pi \int_0^t I_0(t) K_{ab}(r_0) r_0^2 \, dt$$

$$Q_{sc} = \pi \int_0^t I_0(t) K_{sc}(r_0) r_0^2 \, dt$$

$$Q_{ev} = 4\pi \int_0^t j_m L(T_0) r_0^2 \, dt$$

$$Q_T = 4\pi \int_0^t j_T r_0^2 \, dt \tag{26}$$

where the factor of the effectiveness of radiation scattering by the particle $K_{sc}(r_0)$ is determined as in reference [18]. Also, the energy response parameters M_i, $i = 1, 2, 3$ will be considered which characterize the contribution of separate processes into the energy balance of the evaporation process

$$M_1 = \frac{Q_{ev}}{Q_{ev} + Q_T}, \quad M_2 = \frac{Q_{ev}}{Q_{ab} + Q_{sc}}, \quad M_3 = \frac{Q_{ab}}{Q_{ab} + Q_{sc}}. \tag{27}$$

The parameters M_1 and M_3 respectively characterize the contributions of evaporation and absorption into the total energy balance of the evaporating particle which absorbs and scatters the incident radiation energy. The parameter M_2 characterizes the efficiency of the evaporation process.

In energy balance equation (3) the thermal energy of particle, $E_T(t)$, is

$$E_T = c_2 \bar{n}_0 V_0 T_0. \tag{28}$$

Its integration over the time from $t = 0$ to t with regard for equation (26) and the energy Q_y

$$Q_y = 4\pi \int_0^t j_m \{L(T_0) + c_2 T_0\} r_0^2 \, dt \tag{29}$$

will give the energy conservation law for the particle

$$Q_{ab} + E_{T\infty} = E_T(t) + Q_y + Q_T, \tag{30}$$

where $E_{T\infty} = c_2 \bar{n}_0 \frac{4}{3} \pi r_\infty^3 T_\infty$ is the initial thermal energy of the particle. In numerical calculation of the system of equations (3), control of the energy conservation law (30) was performed which was satisfied with an error of $\lesssim 0.1\%$.

The above system of equations can be used for the investigation of the processes of heating and evaporation of spherical particles and droplets exposed to optical radiation with different wavelengths λ.

3. NUMERICAL RESULTS AND DISCUSSION

Consider evaporation, in atmospheric air, of a water droplet of initial radius r_∞, which varies over the range $2.5 \leqslant r_\infty \leqslant 50$ μm, and of initial temperatures $T_\infty = 273$ and 293 K on exposure to continuous optical radiation with the wavelength $\lambda = 10.6$ μm. Except for otherwise specified cases, it will be assumed that the evaporation (condensation) coefficient for water is $\alpha = 1$ and indices of reflection, n_λ, and absorption, κ_λ, for water at the wavelength $\lambda = 10.6$ μm are: $n_\lambda = 1.173$, $\kappa_\lambda = 8.23 \times 10^{-2}$ [18]. The parameters p_1 and b in the temperature dependences of κ_1 and D, equation (8), are assumed to be equal to $p_1 = 0.80$, $b = 1.80$ [15], with p_1 being equal to $b - 1$. The discussion will be restricted to the diffusional regime of evaporation, which is of greatest practical interest, with j_m being defined by equation (12) and j_ε, by equations (22) and (10). The dependence of the evaporation heat L of a water molecule and of the saturated steam density n_0 on T_0 were determined as in reference [15]. The characteristic relaxation times of the inner, t_T and t_c, and outer, t_T^1 and t_d, temperature and density fields for air-suspended water droplets with the initial radius $r_\infty = 2.5$ μm are: $t_T \approx 4.3 \times 10^{-4}$ s, $t_c \approx 3.5 \times 10^{-5}$ s, $t_T^1 \approx t_d \approx 3 \times 10^{-6}$ s and with the radius $r_\infty = 50\,\mu$m: $t_T \approx 1.7 \times 10^{-2}$ s, $t_c \approx 1.4 \times 10^{-3}$ s, $t_T^1 \approx t_d \approx 1.2 \times 10^{-4}$ s. Comparison of these values with those presented in Fig. 5 indicates that the characteristic relaxation times are generally much smaller than the characteristic time of water evaporation, so that the approximations used are valid. In the conditions considered the radiative cooling of water droplets can be neglected [2].

Consider the heating and evaporation of water droplets exposed to continuous radiation with a constant energy flux density $I_0 = 5 \times 10^6$ W m^{-2} Figures 1–6 present some calculated results for

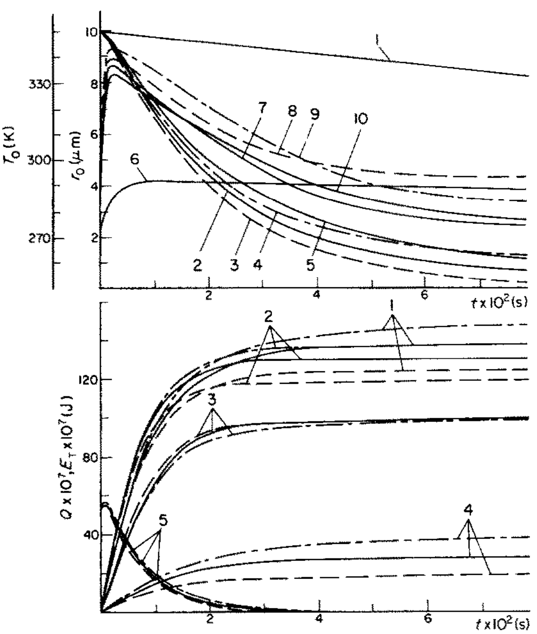

FIG. 1. The time (t) dependence of radius r_0 (1–5) and temperature T_0 (6–10) for $r_\infty = 10\,\mu$m at $I_0 = 5 \times 10^5$ W m^{-2}, $T_\infty = 273$ K, $\alpha = 1$, n_λ, κ_λ [18] (1, 6); at $I_0 = 5 \times 10^6$ W m^{-2}, $T_\infty = 273$ K, $\alpha = 1$, n_λ, κ_λ [18] (2, 7); $T_\infty = 293$ K, $\alpha = 1$, n_λ, κ_λ [18] (3, 8); $T_\infty = 273$ K, $\alpha = 3.6 \times 10^{-2}$, n_λ, κ_λ [18] (4, 9); $T_\infty = 273$ K, $\alpha = 1$, n_λ, κ_λ [26] (5, 10); and the time dependence of the energies Q_{ab}(1), Q_{sc}(2), Q_{ev}(3), Q_T(4) and E_T(5) at $I_0 = 5 \times 10^6$ W m^{-2}, n_λ, κ_λ [18], $\alpha = 1$, $T_\infty = 273$ K (——), $T_\infty = 293$ K (- - - -), $\alpha = 3.6 \times 10^{-2}$, $T_\infty = 273$ K (-·-·-).

evaporation of water droplets. Figure 1 shows the time-dependence of r_0, T_0 and of energies Q, equation (26), and E_T, equation (28), for a droplet of $r_\infty = 10\,\mu$m at the temperature $T_\infty = 273$ K. From the instant the droplet is exposed to continuous radiation, it starts to absorb the radiation energy and its temperature rapidly increases (energy release is much in excess of energy losses). This leads to droplet evaporation and to an increase in the conduction and evaporation heat losses by the droplet. At a certain time t_m, which depends on the initial parameters of the problem, the energy release in the droplet and energy losses due to evaporation and heat conduction balance out and the droplet temperature attains the maximum value T_m. In the process of temperature rise and evaporation, over the time interval $[0, t_m]$, the droplet acquires, by the time t_m, the radius $r_m < r_\infty$. At the time $\sim t_m$ the droplet evaporation rate is at a maximum. In this case $t_m = 2.06 \times 10^{-3}$ s, $T_m = 337.1$ K, $r_m = 9.55\,\mu$m. With T_m having been attained, the temperature of the droplet starts to fall in the process of evaporation at $t > t_m$ (energy release is less than energy losses). As r_0 decreases with time (especially at $t > t_m$), the rate of droplet evaporation also decreases. The characteristic time interval of evaporation, to be used for the assessment of the process duration, is the time of decrease in the initial droplet radius r_∞ down to a certain value, e.g. $t_2(r_0 = r_\infty/2)$ or $t_{10}(r_0 = r_\infty/10)$, i.e. the time of a two- or ten-fold decrease of r_∞ due to evaporation. In this case $t_2 = 1.66 \times 10^{-2}$ s, $t_{10} = 6.08 \times 10^{-2}$ s. In the process of droplet evaporation, Q_{ab}, Q_{sc}, Q_{ev}, Q_y and Q_T increase monotonously, while E_T decreases monotonously with time. Asymptotically when $t \to \infty$ (virtually, when the droplet radius $r_0 \lesssim 0.1 r_\infty$), these energies acquire the constant values $\bar{Q}_{ab}$, $\bar{Q}_{sc}$, $\bar{Q}_{ev}$, $\bar{Q}_y$, $\bar{Q}_T$ and $E_T \approx 0$, i.e. the processes of energy absorption and release by the droplet actually terminate. Then

$$\bar{Q}_{ab} + E_{T\infty} = \bar{Q}_y + \bar{Q}_T. \quad (31)$$

This is because of r_0 decreasing to such an extent in the process of evaporation that the droplet practically ceases to absorb and scatter the radiation energy. The energy release by the droplet decreases sharply because of a significant reduction of its radius and because of the approach of T_0 to the surrounding medium temperature T_∞.

Figure 2 gives the plots of the parameters M_i, equation (27), and D_1, equation (25), vs. the time for the droplet of $r_\infty = 10\,\mu$m at $T_\infty = 273$ K. The parameters M_1 and M_2 depend on the time in the process of droplet evaporation, with M_1 having the maximum at $t \gtrsim t_m$. The parameter M_3 changes little during droplet evaporation. Asymptotically, $M_i (i = 1, 2, 3)$ acquire the constant values $\bar{M}_i$.

The process of droplet evaporation over the time interval $[0, t_m]$ is a substantially non-stationary one and takes place in the conditions of a marked excess of energy release over energy losses at $D_1 > 1$. At the time t_m, the energy release in the droplet and energy losses due to evaporation and heat conduction balance out and $D_1(t_m) = 1$. When $t > t_m$, the energy losses start to exceed the energy release and, at a certain time instant,

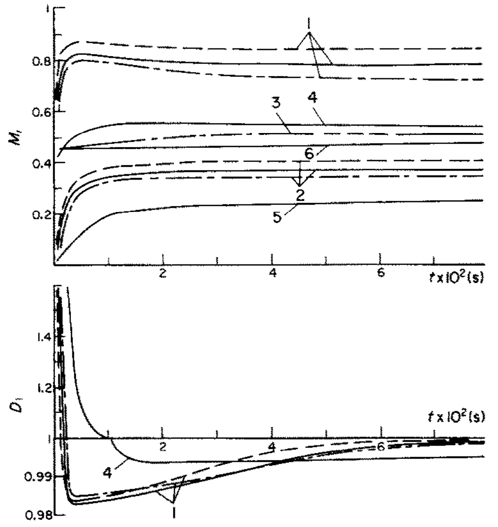

FIG. 2. The time (t) dependence of the parameters M_i, $i = 1$ (1, 4), 2 (2, 5), 3 (3, 6) and D_1 (1, 4) for a droplet with $r_\infty = 10\,\mu$m at $I_0 = 5 \times 10^6$ W m^{-2}, $\alpha = 1$, $T_\infty = 273$ K (——); $T_\infty = 293$ K (- - - -), $\alpha = 3.6 \times 10^{-2}$, $T_\infty = 273$ K (-·-·-) (1–3) and at $I_0 = 5 \times 10^5$ W m^{-2}, $\alpha = 1$, $T_\infty = 273$ K (4–6).

the imbalance between these attains the maximum, while D_1 acquires the minimum value equal in the present case to 0.983. In the course of the process, the rate of evaporation, while regulating the balance between the energy release and energy losses, "keeps watch" on r_0 and T_0 and automatically adjusts itself so that there is some excess of energy losses over the energy release. In this case, T_0 and the amount of imbalance between the energy losses and energy release decrease with time, and D_1 tends to unity. The process of evaporation approaches the quasistationary regime when the energy release and energy losses exactly compensate one another and $D_1 = 1$.

Consider the effect of the initial parameters of the problem, I_0 and T_∞, on the dynamics of droplet evaporation. A decrease of I_0 to $I_0 = 5 \times 10^5$ W m^{-2} at $T_\infty = 273$ K (see Figs 1 and 2) leads to a sharp decrease in the maximum temperature $T_m = 292.1$ K and in droplet evaporation rate, to an increase of t_m and of characteristic times of evaporation t_2, t_{10}. A noticeable increase is observed in $\bar{Q}_{ab}$, $\bar{Q}_{sc}$, $\bar{Q}_T$ (Table 1), in the minimum value of D_1, equal to 0.994; the parameters $\bar{M}_1$ and $\bar{M}_2$ decrease. A decrease in T_m leads to a substantial increase in the heat conduction contribution to the energy balance of the evaporating droplet (M_1, Q_T).

The growth of T_∞ up to $T_\infty = 293$ K at $I_0 = 5 \times 10^6$ W m^{-2} (see Figs 1 and 2) leads to a reduction of the heat conduction contribution into the energy balance and to a higher rate of droplet evaporation. This, in turn, leads to some increase of T_m ($r_\infty = 10\,\mu$m) $= 340.9$ K, of $\bar{M}_1$ and $\bar{M}_2$ and to a decrease of $\bar{Q}_{ab}$, $\bar{Q}_{sc}$ and $\bar{Q}_T$. The effect of I_0 and T_∞ on $\bar{Q}_{ev}$ and $\bar{M}_3$ is insignificant. A similar behaviour of heating and evaporation of water droplets is observed for different r_∞, I_0 and T_∞.

For purposes of comparison with the experimental results [24], numerical calculations have been made for the process of evaporation of water droplets with $r_\infty = 15$, 25 and 35 μm under continuous radiation at $\lambda = 10.6\,\mu$m and at constant $I_0 = 9.3 \times 10^6$ W m^{-2}. Figure 3 presents the measured and calculated functions $r_0(t)$ which agree fairly well, thus attesting to the validity of the model used. It should be noted that in the process of heating and evaporation of droplets, the maximum temperature T_m of droplets with $r_\infty = 25.35\,\mu$m reaches, in this case, the water boiling temperature T_b. Within this temperature region, $T_0 \sim T_b$, it is necessary, along with the diffusional vapour transport from the droplet, equations (12) and (22), to take into account the motion of vapour–air medium. A good agreement between the predicted and experimental functions $r_0(t)$ indicates that the diffusional regime of evaporation with the actual dependences of the diffusion and heat conduction coefficients describe, with a sufficient accuracy, the evaporation of droplets over the temperature range $T_\infty < T_0 \lesssim T_b$.

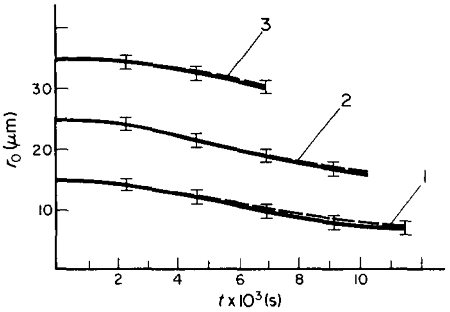

FIG. 3. The dependence of the droplet radius r_0 on time t at $r_\infty = 15$ (1), 25 (2), 35 (3) μm, $I_0 = 9.3 \times 10^6$ W m^{-2}. (——), Experimental data [24], (– – – –), calculation; vertical segments, experimental error.

Consider the effect of the initial parameters of the problem on the evaporation process characteristics for droplets with r_∞ within the range $2.5 \leqslant r_\infty \leqslant 50\,\mu$m. Numerical calculations of the process of water droplet evaporation were performed for $I_0 = 5 \times 10^5$ and 5×10^6 W m^{-2}, $T_\infty = 273$ and 293 K.

Table 1. Parameters of the droplet evaporation process at $\alpha = 1$, $n_\lambda = 1.173$ and $\kappa_\lambda = 8.23 \times 10^{-2}$

I_0 (W m^{-2})	r_∞ (μm)	$\bar{Q}_{ab} \times 10^{-7}$ (J) 273 K	$\bar{Q}_{ab} \times 10^{-7}$ (J) 293 K	$\bar{Q}_{sc} \times 10^{-7}$ (J) 273 K	$\bar{Q}_{sc} \times 10^{-7}$ (J) 293 K	$\bar{Q}_{ev} \times 10^{-7}$ (J) 273 K	$\bar{Q}_{ev} \times 10^{-7}$ (J) 293 K	$\bar{Q}_T \times 10^{-7}$ (J) 273 K	$\bar{Q}_T \times 10^{-7}$ (J) 293 K
5×10^5	2.5	3.79	2.36	0.61	0.38	1.60	1.58	2.13	0.79
	5	28.7	18.7	13.6	8.93	13.0	12.8	15.7	5.98
	10	202	145.1	189.6	137.6	102.7	101.3	96.4	41.9
	20	1357	1095	1690	1370	813	805.3	503	261.3
	30	4180	3572	4830	4110	2720	2705	1590	745
	40	9420	8245	9910	8667	6415	6371	2515	1534
	50	17800	15800	18000	16000	12500	12400	4210	2685
5×10^6	2.5	3.17	2.26	0.50	0.36	1.61	1.59	1.52	0.65
	5	20.5	16.8	9.55	7.99	12.7	12.6	7.15	3.8
	10	137.2	124.2	130	118.6	99.2	98.8	28.3	18.4
	20	997	943.5	1243	1180	776	773.6	112	83.8
	30	3278	3143	3730	3579	2588	2585	264	207.8
	40	7680	7374	7982	7679	6083	6066	493	397.9
	50	14900	14300	15000	14400	11800	11800	809	666

Figure 4 presents the dependence of T_m, T_2 and T_{10} on r_∞ at the time when the evaporating droplets attain the radii $r_0 = r_\infty/2$ and $r_0 = r_\infty/10$ [$T_2 = T_0(r_0 = r_\infty/2)$, $T_{10} = T_0(r_0 = r_\infty/10)$]. Several versions of calculations were employed. The values of T_m for the droplets with $r_\infty \sim 40$–50 μm at $I_0 = 5 \times 10^6\,\mathrm{W\,m^{-2}}$ somewhat exceed the water boiling temperature T_b; however, as the comparison with the experimental data has shown, this does not entail a noticeable error in the determination of the evaporation process parameters for these droplets. An increase in I_0, T_∞ and r_∞ leads to a noticeable increase in the temperatures T_m, T_2 and T_{10}. Figure 4 allows one to find the temperature intervals within which droplets with different r_∞ evaporate in diffusional regime depending on I_0 and T_∞. The minimum values of D_1 for the given values of r_∞, I_0 and T_∞ vary within the range from about 0.98 to 0.99.

It is of interest to assess the role of the non-stationary stage in the process of droplet evaporation, i.e. to compare the time t_m and radius $r_m(t_m)$ with the characteristic time of droplet evaporation and r_∞ for the given values of I_0 and T_∞.

Figure 5 presents the dependence of t_m, t_2 and t_{10} [$r_0(t_2) = r_\infty/2$, $r_0(t_{10}) = r_\infty/10$] on r_∞ for several versions of calculations. The vertical strokes on the curves $t_m(r_\infty)$ denote the values of the droplet radius $r_m(t_m)$. The values of t_m increase monotonously with r_∞; t_2 and t_{10} have a minimum at $r_\infty \sim 10\,\mu$m, which is due to a sharp increase in the contribution of heat conduction at $r_\infty < 10\,\mu$m. The growth of I_0 from 5×10^5 to $5 \times 10^6\,\mathrm{W\,m^{-2}}$ leads to a noticeable decrease of t_m, t_2 and t_{10}. The increase of T_∞ from 273 to 293 K leads to a slight decrease in t_m, t_2 and t_{10}. The growth of r_∞ and I_0 substantially increases the duration of the non-stationary stage of t_m in the overall droplet evaporation process characterized by the times t_2 and t_{10}. In this case, depending on r_∞ and I_0, $0.02t_2 \lesssim t_m \lesssim 0.24t_2$, $0.01t_{10} \lesssim t_m \lesssim 0.12t_{10}$. The increase of r_∞ and I_0 leads to a noticeable increase in the mass of the droplet, which has been evaporated by the time t_m, and to a decrease of r_m, with the latter lying in the region $0.93\,r_\infty \lesssim r_m \lesssim 0.99\,r_\infty$. Consequently, a decrease in the droplet mass for the time t_m may amount to about 20% of the initial droplet mass, and this should be taken into account when describing the evaporation process of a droplet exposed to radiation.

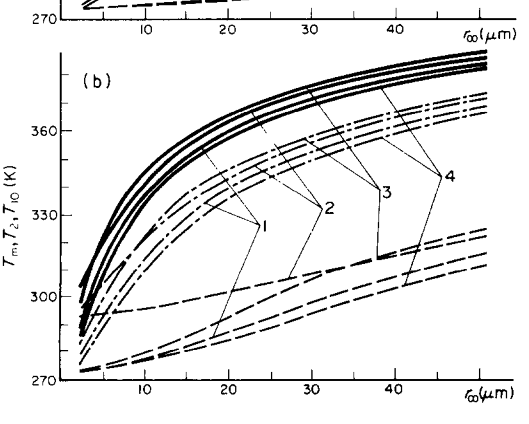

FIG. 4. The dependence of temperature T_m (——), T_2 (–·–·–), T_{10} (– – – –) on r_∞ at $I_0 = 5 \times 10^5\,\mathrm{W\,m^{-2}}$, n_λ, κ_λ[18]; $\alpha = 1$, $T_\infty = 273$ K (1), 293 K (2); $\alpha = 3.6 \times 10^{-2}$, $T_\infty = 273$ K (3) (Fig. 4(a)) and at $I_0 = 5 \times 10^6\,\mathrm{W\,m^{-2}}$, n_λ, κ_λ[18]; $\alpha = 1$, $T_\infty = 273$ K (1), $T_\infty = 293$ K (2); $\alpha = 3.6 \times 10^{-2}$, $T_\infty = 273$ K (3), n_λ, κ_λ [26], $\alpha = 1$, $T_\infty = 273$ K(4) (Fig. 4(b)).

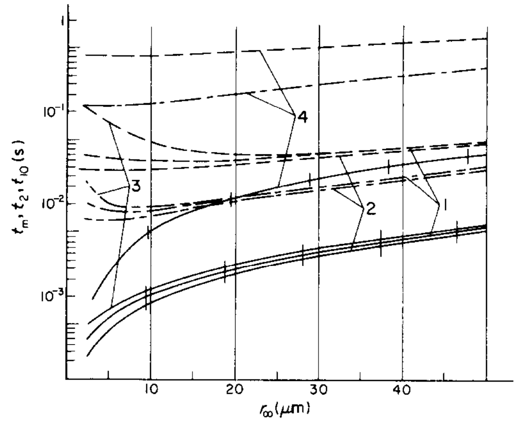

FIG. 5. The dependence of times t_m (——), t_2 (–·–·–), t_{10} (– – – –) on r_∞ at $I_0 = 5 \times 10^6\,\mathrm{W\,m^{-2}}$, $\alpha = 1$, $T_\infty = 273$ K (1) and $T_\infty = 293$ K (2); $\alpha = 3.6 \times 10^{-2}$, $T_\infty = 273$ K (3); $I_0 = 5 \times 10^5\,\mathrm{W\,m^{-2}}$, $\alpha = 1$, $T_\infty = 273$ K (4).

Table 1 contains the calculated values of $\bar{Q}_{ab}$, $\bar{Q}_{sc}$, $\bar{Q}_{ev}$ and $\bar{Q}_T$ for droplets at $I_0 = 5 \times 10^5$ and $5 \times 10^6\,\mathrm{W\,m^{-2}}$ and $T_\infty = 273$ and 293 K. The energies $\bar{Q}_{ab}$, $\bar{Q}_{sc}$, $\bar{Q}_{ev}$ and $\bar{Q}_T$ depend substantially on I_0, r_∞ and T_∞. The increase of r_∞ by an order of magnitude at constant I_0 and T_∞ leads to an increase in $\bar{Q}_{ab}$, $\bar{Q}_{sc}$, $\bar{Q}_{ev}$ by about 10^3 times and in $\bar{Q}_T$ by about 10^2–10^3 times. The increase of I_0 at constant r_∞ and T_∞ leads to a noticeable decrease of $\bar{Q}_{ab}$, $\bar{Q}_{sc}$ and especially of $\bar{Q}_T$. This is attributed to a more rapid droplet evaporation which reduces $\bar{Q}_{ab}$, $\bar{Q}_{sc}$ and to a decrease in the heat conduction contribution, $\bar{Q}_T$, to the energy balance of the evaporating droplet. The rise of T_∞ at constant I_0 and r_∞ leads to a noticeable decrease of $\bar{Q}_{ab}$, $\bar{Q}_{sc}$, $\bar{Q}_T$ due to a smaller heat conduction contribution into the energy balance of evaporation. The energy $\bar{Q}_{ev}$ depends weakly on I_0 and T_∞.

Figure 6 presents the dependences of the stationary parameters $\bar{M}_i$ and the effectiveness factors of radiation absorption, K_{ab}, and scattering, K_{sc}, at $\lambda = 10.6\,\mu$m by a droplet of radius from r_∞ to r_0. The parameter $\bar{M}_1$, equation (27), increases with I_0, r_∞ and T_∞ due to a

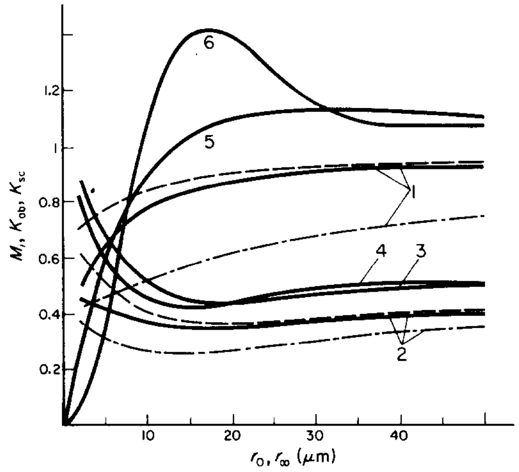

FIG. 6. The dependence of parameters $\bar{M}_i$, $i = 1$ (1), 2 (2), 3 (3), and of $\tilde{M}_3$ (4) at $I_0 = 5\times10^6$ W m^{-2}, $T_\infty = 273$ K (——) and $T_\infty = 293$ K (----) and $I_0 = 5\times10^5$ W m^{-2}, $T_\infty = 273$ K (-·-·-) and of factors K_{ab} (5) and K_{sc} (6) on r_∞ and r_0.

decrease in $\bar{Q}_T$. The parameter M_2, equation (27), increases with I_0 and T_∞. $\bar{M}_2$ and $\bar{M}_3$ have the minimum at $r_\infty \sim 10$–25 μm which can be explained by the maximum of the effectiveness factor of scattering K_{sc} in the specific range of r_∞. $\bar{M}_3$ does not practically depend on I_0 and T_∞ (in Fig. 6 the dependences of $\bar{M}_3$ on r_∞ at $I_0 = 5\times10^5$ and 5×10^6 W m^{-2}, $T_\infty = 273$ and 293 K practically coincide) and changes little in the process of droplet evaporation (see Fig. 2). This indicates that $\bar{M}_3$ may be governed by the initial problem parameters and is independent of the process evaporation dynamics. It will be assumed approximately that

$$\tilde{M}_3(r_\infty) = \frac{K_{ab}(r_\infty)}{K_{ab}(r_\infty)+K_{sc}(r_\infty)}. \tag{32}$$

The function $\tilde{M}_3(r_\infty)$ is given in Fig. 6. Expression (32) correctly describes the dependence of $\bar{M}_3$ on r_∞, agrees satisfactorily with the calculated values of $\bar{M}_3$ and can be used for an approximate evaluation of $\bar{M}_3$.

Consider the effect of the evaporation coefficient α and of n_λ, κ_λ for water on the process of evaporation. The values α and n_λ, κ_λ at $\lambda = 10.6$ μm measured in different experiments differ noticeably. According to recent experiments [25], the most exact seems to be the value $\alpha = 1$ which was used above. However, in order to elucidate the effect of α on the parameters of droplet evaporation, the calculations at the lowest knowable value of $\alpha = 3.6\times10^{-2}$ [25] and at former $n_\lambda = 1.173$, $\kappa_\lambda = 8.23\times10^{-2}$ [18] have been carried out, the results of which are presented in Figs 1, 2, 4 and 5 and listed in Table 2. With $\alpha = 3.6\times10^{-2}$, the droplet evaporation is slower and the droplet temperature increases by about 2–10 K. This leads to a noticeable increase in $\bar{Q}_{ab}$, $\bar{Q}_{sc}$ and $\bar{Q}_T$, and to a corresponding change in $\bar{M}_1$ and $\bar{M}_2$, while $\bar{Q}_{ev}$ and $\bar{M}_3$ change insignificantly. Consequently, a change in α may appreciably influence the parameters of the droplet evaporation process. In order to find out the influence of n_λ, κ_λ on droplet evaporation, calculations were carried out at $\alpha = 1$ and at the least knowable values of $n_\lambda = 1.144$, $\kappa_\lambda = 6.7\times10^{-2}$ [26] for $\lambda = 10.6$ μm. The function $K_{ab}(r_0)$ was determined from expression (4) (see, for example, [11]). The results of calculations are presented in Figs 1 and 4 and listed in Table 2. For $n_\lambda = 1.144$, $\kappa_\lambda = 6.7\times10^{-2}$ [26], the maximum temperature of droplets T_m decreases by about 2–5 K, since $K_{ab}(r_0)$ in this case is

Table 2. Parameters of the droplet evaporation process

$T_\infty = 273$ K, $\alpha = 3.6\times10^{-2}$, $n_\lambda = 1.173$, $\kappa_\lambda = 8.23\times10^{-2}$

I_0 (W m^{-2})	r_∞ (μm)	$\bar{Q}_{ab}\times10^{-7}$ (J)	$\bar{Q}_{sc}\times10^{-7}$ (J)	$\bar{Q}_{ev}\times10^{-7}$ (J)	$\bar{Q}_T\times10^{-7}$ (J)	$\bar{M}_1$	$\bar{M}_2$	$\bar{M}_3$
5×10^6	2.5	5.94	0.827	1.59	4.34	0.270	0.235	0.879
	5	26.5	11.43	12.52	13.1	0.489	0.330	0.699
	10	148.3	137.1	98.2	38.9	0.717	0.344	0.520
	20	1020	1264	771	130.7	0.856	0.337	0.446
	30	3315	3764	2576	293	0.899	0.364	0.468
	40	7730	8030	6061	528	0.920	0.384	0.490
	50	1.5×10^4	1.51×10^4	1.18×10^4	852	0.933	0.392	0.499

$T_\infty = 273$ K, $\alpha = 1$, $n_\lambda = 1.144$, $\kappa_\lambda = 6.7\times10^{-2}$

I_0 (W m^{-2})	r_∞ (μm)	$\bar{Q}_{ab}\times10^{-7}$ (J)	$\bar{Q}_{sc}\times10^{-7}$ (J)	$\bar{Q}_{ev}\times10^{-7}$ (J)	$\bar{Q}_T\times10^{-7}$ (J)	$\bar{M}_1$	$\bar{M}_2$	$\bar{M}_3$
5×10^6	2.5	3.28	0.477	1.61	1.63	0.497	0.431	0.873
	5	21.33	8.59	12.72	8.01	0.614	0.425	0.713
	10	140.1	122.9	99.6	31.5	0.759	0.379	0.533
	20	1005	1289	778	120.9	0.865	0.339	0.438
	30	3289	4071	2593	277.5	0.903	0.352	0.447
	40	7689	8450	6089	509.7	0.923	0.377	0.476
	50	1.49×10^4	1.54×10^4	1.18×10^4	823.6	0.935	0.389	0.492

somewhat smaller than $K_{ab}(r_0)$ for n_λ, κ_λ as in reference [18], and the evaporation rate decreases noticeably. This also leads to a change in $\bar{Q}_{ab}$, $\bar{Q}_{sc}$, $\bar{Q}_T$, $\bar{M}_1$, $\bar{M}_2$ and $\bar{M}_3$. The quantity $\bar{Q}_{ev}$ changes insignificantly. Thus, the change in n_λ, κ_λ may appreciably influence the rate of evaporation and the parameters of the process.

4. SIMPLE MODEL OF PARTICLE EVAPORATION. NUMERICAL AND ANALYTICAL SOLUTIONS

4.1. *Non-stationary evaporation*

In the absence of temperature and density jumps at the surface of an evaporating particle and at constant heat conduction, $\kappa_{1\infty}$, and diffusion, D_∞, coefficients and heat of evaporation L_∞, the process of particle evaporation is described by the system of equations (3), with equation (16) taken into account,

$$\bar{n}_0 c_2 V_0 \frac{\mathrm{d}T_0}{\mathrm{d}t} = \tfrac{1}{4} I_0(t) K_{ab} S_0 - \frac{D_\infty}{r_0}(n_0 - n_\infty) L_\infty S_0 - \frac{\kappa_{1\infty}}{r_0}(T_0 - T_\infty) S_0 \quad (33)$$

$$\bar{n}_0 \frac{\mathrm{d}V_0}{\mathrm{d}t} = -\frac{D_\infty}{r_0}(n_0 - n_\infty) S_0 \quad (34)$$

with the initial conditions

$$r_0(t=0) = r_\infty, \quad T_0(t=0) = T_\infty \quad (35)$$

where the values of D_∞, $\kappa_{1\infty}$, L_∞ are taken at T_∞. In order to find out the effect of jumps and temperature dependence of D, κ_1 and L [15], numerical calculations of equations (33)–(35) have been carried out at different I_0, T_∞ and r_∞. The magnitude of the vapour density jump at the surface of an evaporating droplet, $\Delta n = (n_0 - \bar{n})/n_0 \times 100\%$, at the given I_0, r_∞, T_∞ and $\alpha = 1$ varies within the range $1 \lesssim \Delta n \lesssim 6\%$. At $\alpha = 3.6 \times 10^{-2}$, the jump Δn changes in the range $10 \lesssim \Delta n \lesssim 50\%$, i.e. a decrease in α leads to a sharp increase in Δn. Besides, Δn depends on time in the process of droplet evaporation. The difference of temperatures $\Delta T = T_0 - \bar{T}$ on the surface and near the surface of the droplet amounts to $0.5 \lesssim \Delta T \lesssim 4$ K, with ΔT decreasing in the process of evaporation and being weakly dependent on α. Since the quantities $\bar{T}$ and $\bar{n}$ enter into expressions for j_T, equation (10), and for j_m, equations (12) and (14), then the importance of the allowance for the vapour density and temperature jumps at the surface of an evaporating droplet is apparent. It should be noted that the allowance for Δn is especially important at $\alpha = 3.6 \times 10^{-2}$. Droplets evaporate in the temperature range $T_\infty \leqslant T_0 \leqslant T_m$ where the maximum temperature T_m depends on I_0, r_∞, T_∞ and α, n_λ, κ_λ (see Fig. 4). The inclusion of the temperature dependence of κ_1, D, L {see equation (8) and reference [15]} over the interval $T_m - T_\infty$ leads to an increase of κ_1, D and to a decrease of L by about 10–60%. Thus, the magnitude of the error incurred by the replacement of the actual temperature dependences of κ_1, D and L {see equation (8) and reference [15]} by constant coefficients at T_∞ depends substantially on I_0, r_∞ and T_∞, and may be appreciable. Characteristic of a simple model of droplet evaporation, equations (33) and (34), is the overestimation of the maximum temperature T_m and of temperature T_0 in the process of evaporation by about 1–6 K, with this overestimation decreasing with time. In spite of this, the use of equations (33) and (34) leads to underestimation of the droplet evaporation rate. This, in turn, leads to overestimation of the energies $\bar{Q}_{ab}$, $\bar{Q}_{sc}$ and $\bar{Q}_{ev}$ by about 2–10%, and of $\bar{Q}_T$ by about 10–1% at $\alpha = 1$, and to a change in the energies $\bar{Q}_{ab}$ and $\bar{Q}_{sc}$ by -50 to $+10\%$, to overestimation of $\bar{Q}_{ev}$ by 2–10%, underestimation of $\bar{Q}_T$ by 60–5% at $\alpha = 3.6 \times 10^{-2}$, respectively for the radii lying in the range $r_\infty = 2.5$–50 μm. Consequently, the use of the simple droplet evaporation model, equations (33) and (34), for $r_\infty \leqslant 10$ μm, $I_0 \lesssim 1 \times 10^6$ W m^{-2} at $\alpha = 1$ leads to errors of about 5–10% in the determination of evaporation parameters. The use of equations (33) and (34) for $r_\infty > 10$ μm, $I_0 > 1 \times 10^6$ W m^{-2} and especially at the condensation coefficient $\alpha = 3.6 \times 10^{-2}$ may lead to errors of about 10–50% in the determination of the dynamic and energy response parameters of droplet evaporation.

For droplets of small radius, $r_0 < \lambda$, the effectiveness factor of radiation absorption by a droplet, K_{ab}, is approximately equal to: $K_{ab} \approx \bar{k}_{ab} r_0$ [see equation (4)]. With allowance for this circumstance, the system of equations (33) and (34) together with equation (35) can be represented as

$$\frac{\mathrm{d}y}{\mathrm{d}\tau} = 1 - \frac{y - y_\infty}{x^2} + \frac{1}{3x}\frac{\mathrm{d}x}{\mathrm{d}\tau}$$

$$x\frac{\mathrm{d}x}{\mathrm{d}\tau} = -\mu\left(\mathrm{e}^{-1/y} - \mathrm{e}^{-1/y_\infty}\right) \quad (36)$$

$$y(0) = y_\infty;\ x(0) = x_\infty;\ y = \frac{kT_0}{L_\infty};\ x = \frac{r_0}{r_*},\ \tau = \frac{t}{t_*}$$

$$r_* = \left(\frac{4\kappa_{1\infty} L_\infty}{I_0 \bar{k}_{ab} k}\right)^{1/2};\ t_* = \frac{12\bar{n}_0 L_\infty}{I_0 \bar{k}_{ab}};\ \mu = \frac{3AkD_\infty}{\kappa_{1\infty}}$$

where the saturated steam density is determined from

$$n_0 = A\,\mathrm{e}^{-L_\infty/kT_0} \quad (37)$$

and it is assumed that $c_2 = 9$ K. The results of three versions of numerical calculation of equation (36) are presented in Fig. 7. Here, the time τ is reckoned from the time of y_m maximum attainment so that the total time of the process is equal to the sum of the left- and right-hand side abscissas. A remarkable property of the system of equations (36) is that the curves $y(\tau)$, $x(\tau)$ have a self-similar character with respect to r_∞ and I_0. The substitution of the numerical values of $\kappa_{1\infty}$, L_∞, k, $\bar{n}_0$, and A into equation (36) gives

$$x = 44.1\, r_0(\bar{k}_{ab} I_0)^{1/2}, \quad \tau = 0.332\, I_0 \bar{k}_{ab} t \quad (38)$$

where I_0 is in W m^{-2}. From Fig. 7 and equation (38), it is possible to find $r_0(t)$ and $T_0(t)$ for wide ranges of I_0 and r_∞, at which evaporation of droplets occurs in diffusional regime.

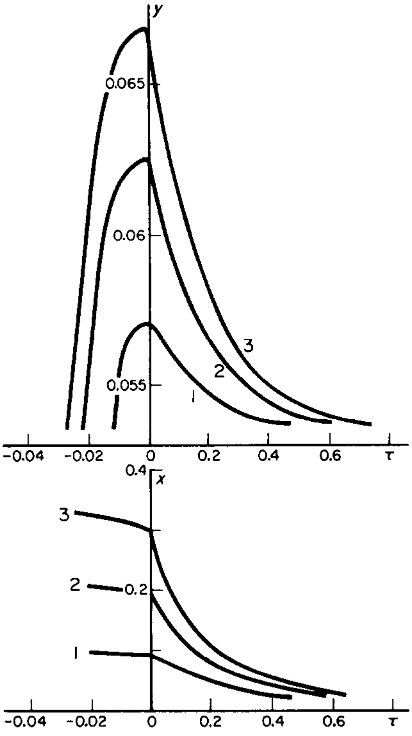

FIG. 7. The dependence of the dimensionless temperature y and of dimensionless droplet radius x on dimensionless time τ. $y_m = 0.0572$, $T_m = 310$ K (1), $y_m = 0.0627$, $T_m = 340$ K (2), $y_m = 0.067$, $T_m = 363$ K (3); the initial temperature $T_\infty = 300$ K ($y = 0.0535$).

The maximum temperature of droplet at $I_0 = \text{const.}$ can be approximately evaluated from equation (33) assuming $\partial T_0/\partial t = 0$ and $r_0(t_m) \approx r_\infty$

$$\frac{1}{4} I_0 K_{ab}(r_\infty) = \frac{D_\infty}{r_\infty}[n_0(T_m) - n_\infty]L_\infty + \frac{\kappa_{1\infty}}{r_\infty}(T_m - T_\infty). \quad (39)$$

In the region of small superheats, $T_0 - T_\infty < 10$ K, equation (39), together with the expansion of $n_0(T_m) - n_\infty$ from equation (37), give the following expression for T_m:

$$T_m = T_\infty + \frac{I_0 K_{ab}(r_\infty) r_\infty k T_\infty^2}{4(D_\infty L_\infty^2 n_\infty + \kappa_{1\infty} k T_\infty^2)}. \quad (40)$$

If T_m is known and the energy losses are neglected, the droplet energy balance equation (33) can be used to estimate the time for the attainment of the temperature maximum

$$t_m = \frac{(T_m - T_\infty) 4\bar{n}_0 c_2 r_\infty}{3 I_0 K_{ab}(r_\infty)}. \quad (41)$$

The values of T_m and t_m obtained from equations (39)–(41) agree satisfactorily with the results of numerical calculations. For example, at $r_\infty = 10\ \mu$m, $T_\infty = 273$ K and $I_0 = 5 \times 10^6$ W m^{-2}, it is possible to obtain from equations (39) and (41) that $T_m \approx 345$ K, $t_m \approx 1 \times 10^{-3}$ s; at $r_\infty = 5\ \mu$m, $I_0 = 5 \times 10^5$ W m^{-2} and $T_\infty = 273$ K, equation (40) yields $T_m \approx 281.6$ K (see Figs 1 and 4).

It is of interest to obtain an analytic solution of the system of equations (33) and (34). However, this is a complex non-linear system of equations and its analytical solution can be obtained only with the use of certain simplifications. Similar to reference [27], it is possible to represent $n_0 - n_\infty$, with allowance for equation (37), as

$$n_0 - n_\infty \approx n_\infty \left[\exp\left(\frac{L_\infty}{k T_\infty^2}(T_0 - T_\infty)\right) - 1\right] \quad (42)$$

thus receiving a good accuracy and simultaneously retaining the characteristic non-linearity typical of the problem. In the region of high superheats, when the droplet energy losses by heat conduction and 1 in equation (42) can be neglected, the system of equations (33) and (34) for droplets with $r_0 < \lambda$, $K_{ab} \approx \bar{\kappa}_{ab} r_0$ and $I_0 = \text{const.}$ has the following analytical solution:

$$r_0 = r_\infty \left[1 + \frac{4 D_\infty n_\infty k c_2 T_\infty^2}{3 r_\infty^2 L_\infty \bar{\kappa}_{ab} I_0 h} \times \left\{\exp\left(\frac{3 L_\infty \bar{\kappa}_{ab} I_0}{4 k \bar{n}_0 c_2 T_\infty^2} t\right) - 1\right\}\right]^{-h} \quad (43)$$

$$T_0 = T_\infty + \frac{3 \bar{\kappa}_{ab} I_0}{4 \bar{n}_0 c_2} t + \frac{3 L_\infty}{c_2} \ln \frac{r_0}{r_\infty}$$

where

$$h = \frac{c_2 T_\infty^2 k}{3 L_\infty^2 - 2 k c_2 T_\infty^2}.$$

For $t \gg 1$, neglecting 1 in equation (43), it is possible to obtain

$$r_0 = r_\infty \left(\frac{3 r_\infty^2 L_\infty \bar{\kappa}_{ab} I_0 h}{4 D_\infty n_\infty k c_2 T_\infty^2}\right)^h \times \exp\left(-\frac{3 L_\infty \bar{\kappa}_{ab} I_0 h}{4 k \bar{n}_0 c_2 T_\infty^2} t\right). \quad (44)$$

The characteristic time of droplet evaporation t_{ev} from equation (44) is

$$t_{ev} = \frac{4 k \bar{n}_0 c_2 T_\infty^2}{3 L_\infty \bar{\kappa}_{ab} I_0 h}. \quad (45)$$

The time dependencies of r_0 and T_0 from equation (43) agree satisfactorily with those calculated by equation (3).

4.2. *Quasistationary evaporation*

Consider the case of quasistationary evaporation when it is assumed that energy release by a particle and energy losses due to evaporation and heat conduction exactly compensate one another, i.e.

$$T_0 = \text{const.} \quad \frac{dT_0}{dt} = 0. \quad (46)$$

Then, equation (33) can be replaced by

$$I_0(t)K_{ab}(r_0)r_0 = 4D_\infty(n_0-n_\infty)L_\infty + 4\kappa_{1\infty}(T_0-T_\infty) = \text{const.} \quad (47)$$

and the quasistationarity parameter from equation (25) is $D_1(r_0) = 1$. Condition (47) is satisfied at the time when t_m reaches T_m at r_m. With equation (47) taken into account, the solution of evaporation kinetics equation (34) at $t \geqslant t_m$ is

$$r_0 = \left[r_m^2 - \frac{2D_\infty}{\bar{n}_0}(n_0-n_\infty)(t-t_m)\right]^{1/2}. \quad (48)$$

The function $I_0(t)$, which provides the fulfilment of the quasistationarity condition (47) together with equation (4), is determined by the relation

$$I_0(t) = \text{const.}/[K_{ab}(r_0)\cdot r_0]. \quad (49)$$

The time of droplet evaporation t_{ev} from equation (48) is

$$t_{ev} = t_m + \frac{r_m^2\bar{n}_0}{2D_\infty(n_0-n_\infty)} \quad (50)$$

with $I_0(t \to t_{ev}) \to \infty$.

At the constant density of radiation energy flux $I_0 = \text{const.}$, the condition $\mathrm{d}T_0/\mathrm{d}t = 0$ is fulfilled only at the time when t_m reaches T_m. At $I_0 = \text{const.}$ and $t > t_m$, the condition $\mathrm{d}T_0/\mathrm{d}t = 0$ cannot be fulfilled, and the following condition holds approximately

$$\frac{1}{4}I_0K_{ab}(r_0) \approx \frac{D_\infty}{r_0}(n_0-n_\infty)L_\infty + \frac{\kappa_{1\infty}}{r_0}(T_0-T_\infty) \quad (51)$$

and it is assumed that temperature varies very slowly in time. Since r_0 decreases due to evaporation, this leads with time to a decrease in T_0, $\mathrm{d}T_0/\mathrm{d}T \lesssim 0$, when condition (51) is fulfilled. The use of the quasistationary condition (51) means that the non-stationary stage of heating the particle from t_m to T_m is neglected, i.e. means an instantaneous heating of droplets at time $t = 0$ up to the quasistationary temperature $T_m(r_\infty, I_0)$, determined from equation (51) at $r_0 = r_\infty$, and an immediate start of intensive evaporation. When the time interval t_m is much smaller than the characteristic time of particle evaporation $t_2(t_{10})$, and the mass loss by a particle for the time t_m is small, as compared with the initial particle mass, then the error incurred by the use of the quasistationary approximation will be small. With the growth of I_0 and r_∞, when the duration of the non-stationary stage t_m and the losses of mass by the particle for the time t_m increase (see Fig. 5), the error introduced by the use of equation (51) will also increase. It is clear that if the quasistationarity condition (51) is applied from the time $t = 0$, the evaporation of the particle will be more rapid and the energies $\bar{Q}_{ab}$, $\bar{Q}_{sc}$ and $\bar{Q}_T$ will be smaller than in the non-stationary case. Equations (34) and (51) were calculated numerically with conditions (35) to elucidate the region of applicability and the accuracy of the quasistationarity condition for the process of droplet evaporation. In the case of quasistationary evaporation the temperature and radius of a droplet at $t > t_m$ are smaller than its temperature and radius in the case of non-stationary evaporation. This leads to the underestimation of energies $\bar{Q}_{ab}$ and $\bar{Q}_{sc}$ by about 1–25%, of $\bar{Q}_T$ by about 1–10% within the range of $r_\infty = 2.5$–50 μm, with $\bar{Q}_{ev}$ remaining virtually intact. The accuracy of the use of the quasistationarity condition improves with a decrease in I_0 and increase in T_∞ within the ranges of their variation given above. Consequently, the use of the quasistationarity condition (51) for $r_\infty \lesssim 10\,\mu$m, $I_0 \lesssim 1\times10^6\,\mathrm{W\,m^{-2}}$ incurs errors of about 5–10% in the determination of the evaporation process parameters; the use of equation (51) for $r_\infty > 10\,\mu$m, $I_0 > 1\times10^6\,\mathrm{W\,m^{-2}}$ may lead in this case to the errors of about 10–30%.

The quasistationary evaporation of droplets on exposure to radiation was the object of investigation in many studies [2, 5, 6, 12, 13]. Equations (34) and (51), together with equation (35), were solved analytically with the use of different simplifications and expressions for $K_{ab}(r_0)$. As an example, the following analytical solution can be given which is obtainable in the region of high superheats, when heat conduction can be neglected:

$$\int_{r_\infty}^{r_0}\frac{\mathrm{d}r_0}{K_{ab}(r_0)} = -\frac{I_0t}{4\bar{n}_0D_\infty}. \quad (52)$$

Thus, on the basis of numerical and analytical solutions of the formulated system of equations, the heating and evaporation in a diffusional regime of a spherical particle exposed to strong optical radiation have been considered in this paper. Note, that the diffusional regime of water droplet evaporation is valid at $p_0 \lesssim 0.5\,p_\infty$, i.e. until $T_0 - T_\infty \lesssim 80$ K and the Stefan flow from the droplet can be neglected.

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THEORIE DU CHAUFFAGE ET DE L'EVAPORATION D'UNE PARTICULE SPHERIQUE EXPOSEE A UN RAYONNEMENT OPTIQUE

Résumé—On développe l'étude théorique du chauffage et de l'évaporation d'une particule sphérique du chauffage et de l'évaporation d'une particule soumise à une fort rayonnement optique. Un système d'évaporation est formulé pour decrire le mécanisme d'évaporation de la particule dans un régime diffusionnel. A partir de la solution numérique de ce système, l'évaporation de gouttelettes d'eau en suspension dans l'air est étudiée en détail. Les valeurs variables et stationnaires des paramètres du mécanisme d'évaporation sont obtenues pour un large domaine des paramètres opératoires initiaux. Les résultats du calcul sont comparés avec les données expérimentales. Un modèle simple de l'évaporation est considéré et des solutions analytiques et numériques sont obtenues.

ERWÄRMUNG UND VERDAMPFUNG EINES KUGELFÖRMIGEN PARTIKELS INFOLGE OPTISCHER BESTRAHLUNG

Zusammenfassung—Erwärmung und Verdampfung eines kugelförmigen Partikels bei starker optischer Bestrahlung wurden theoretisch untersucht. Es wurde ein Gleichungssystem formuliert, welches den Prozeß der Verdampfung im Diffusionsbereich beschreibt. Basierend auf der numerischen Lösung dieses Gleichungssystems, wurde die Verdampfung von in Luft suspendierten Wassertröpfchen unter IR-Strahlung detailliert untersucht. Die zeitabhängigen und stationären Werte der Verdampfungsparameter wurden für einen großen Bereich von Anfangswerten ermittelt. Die berechneten Ergebnisse wurden mit experimentellen Daten verglichen. Es wurde ein einfaches Modell der Verdampfung von Partikeln entwickelt und dafür analytische und numerische Lösungen gefunden.

ТЕОРИЯ НАГРЕВА И ИСПАРЕНИЯ СФЕРИЧЕСКОЙ ЧАСТИЦЫ ПОД ДЕЙСТВИЕМ ОПТИЧЕСКОГО ИЗЛУЧЕНИЯ

Аннотация—В работе проведено теоретическое исследование процессов нагрева и испарения сферической частицы под действием интенсивного оптического излучения. Сформулирована система уравнений, описывающая процесс испарения частицы в диффузионном режиме. С помощью численного решения системы уравнений подробно исследовано испарение капель воды, находящихся в воздухе, под действием инфракрасного оптического излучения. Получены временные зависимости и стационарные значения параметров процесса испарения для широкого диапазона изменения начальных параметров задачи. Проведено сравнение результатов расчетов с экспериментальными данными. Рассмотрена простая модель процесса испарения частицы, получены аналитическое и численное решения.